PRODUCTOS

$\vec{u} = (u_1, u_2, u_3)$ $\quad \vec{v} = (v_1, v_2, v_3)$ $\quad \vec{w} = (w_1, w_2, w_3)$

- ✓ ESCALAR: $\vec{u} \cdot \vec{v} = u_1 \cdot v_1 + u_2 \cdot v_2 + u_3 \cdot v_3$
- ✓ VECTORIAL: $\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$ $\quad \vec{u} \perp \vec{v} \Leftrightarrow \vec{u} \cdot \vec{v} = 0$ $\quad \vec{u} \times \vec{v} \perp \vec{u} \wedge \vec{u} \times \vec{v} \perp \vec{v}$
- ✓ MIXTO: $[\vec{u}, \vec{v}, \vec{w}] = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = \vec{u} \cdot (\vec{v} \times \vec{w})$

ÁREAS Y VOLÚMENES

$A = (a_1, a_2, a_3)$ $\quad B = (b_1, b_2, b_3)$ $\quad C = (c_1, c_2, c_3)$ $\quad D = (d_1, d_2, d_3)$

- ÁREA:
  - ✓ PARALELOGRAMO: $\mathcal{A} = \left|\overrightarrow{AB} \times \overrightarrow{AC}\right| \ u^2$
  - ✓ TRIÁNGULO: $\mathcal{A} = \left|\overrightarrow{AB} \times \overrightarrow{AC}\right|/2 \ u^2$
- VOLUMEN:
  - ✓ PARALEPÍPEDO: $\mathcal{V} = \left|\left[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\right]\right| \ u^3$
  - ✓ TETRAEDRO: $\mathcal{V} = \left|\left[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\right]\right|/6 \ u^3$

GEOMETRÍA ESPACIAL MÉTRICA

notodoesmatematicas.com

youtube.com/jmsreales

ÁNGULOS

- ✓ Dos rectas: $\cos\alpha = \dfrac{|\overrightarrow{v_r} \cdot \overrightarrow{w_s}|}{|\overrightarrow{v_r}||\overrightarrow{w_s}|}$
- ✓ Dos planos: $\cos(90 - \alpha) = \dfrac{|\overrightarrow{v_r} \cdot \overrightarrow{n_\pi}|}{|\overrightarrow{v_r}||\overrightarrow{n_\pi}|}$
- ✓ Recta y plano: $\cos\alpha = \dfrac{\left|\overrightarrow{n_{\pi_1}} \cdot \overrightarrow{n_{\pi_2}}\right|}{\left|\overrightarrow{n_{\pi_1}}\right|\left|\overrightarrow{n_{\pi_2}}\right|}$

DISTANCIAS

$P = (p_1, p_2, p_3)$ $\quad Q = (q_1, q_2, q_3)$

- ✓ Dos puntos: $d(P, Q) = \left|\overrightarrow{PQ}\right|$
- ✓ Punto a plano: $d(P, \pi) = \dfrac{|Ap_1 + Bp_2 + Cp_3 + D|}{\sqrt{A^2 + B^2 + C^2}}$
- ✓ Dos rectas paralelas: $d(r, s) = d(r, B_s) = d(A_r, s)$
- ✓ Recta a plano: $d(r, \pi) = d(A_r, \pi)$
- ✓ Dos planos paralelos: $d(\pi_1, \pi_2) = d\left(\pi_1, B_{\pi_2}\right) = d(A_{\pi_1}, \pi_2)$
- ✓ Punto a recta: $d(P, r) = \dfrac{\left|\overrightarrow{PA_r} \times \overrightarrow{v_r}\right|}{|\overrightarrow{v_r}|}$
- ✓ Dos rectas que se cruzan: $d(r, s) = \dfrac{\left|\left[\overrightarrow{v_r}, \overrightarrow{w_s}, \overrightarrow{A_r B_s}\right]\right|}{|\overrightarrow{v_r} \times \overrightarrow{w_s}|}$